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## Is the Condensation of Strings the Origin of Einstein Gravity ?

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### Abstract

A mechanism of generating the metric is proposed, where the Kalb-Ramond symmetry existing in the topological BF theory is broken through the condensation of the string fields which are so introduced as to couple with the anti-symmetric tensor fields  $B$ , invariantly under the Kalb-Ramond symmetry. In the chiral decomposition of the local Lorentz group, the non-Abelian  $B$  fields need to be generalized to the string fields. The mechanism of the condensation is discussed, viewing the confinement problem and the polymer physics.

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# 1 Introduction

There is an interesting folklore (probably after E. Witten [1]) that at extremely high energy, the concept of the metric disappears and the topological nature of space-time only remains, where all the dynamical degrees of freedom are washed away by the huge symmetries existing in the theory called topological field theory. When we go down to the lower energy, however, due to some unknown phase transition, violation of the huge symmetries occurs and the usual Einstein gravity appears with the recovery of the concept of the metric and of the local degrees of freedom.

We know that the Einstein gravity can be reformulated [2], starting from a topological theory (somebody calls BF theory [3]) written in terms of the anti-symmetric tensor field (2-form field)  $B_{\mu\nu}^{AB}(x)$  and the Riemann tensor  $R_{\lambda\rho}^{AB}(x) = \partial_\lambda \omega_\rho^{AB}(x) - \partial_\rho \omega_\lambda^{AB}(x) + \omega_\lambda^{AC}(x)\omega_\rho^{CB}(x) - \omega_\rho^{AC}(x)\omega_\lambda^{CB}(x)$ , given by the spin connection  $\omega_\mu^{AB}(x)$ . The action of this theory reads

$$S = \int \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} B_{\mu\nu}^{AB}(x) R_{\lambda\rho}^{AB}(x), \quad (1)$$

where we have used the Euclidean metric for the local Lorentz frame, namely  $A, B$  and  $C$  take  $1 \sim 4$ . If we fix the anti-symmetric tensor field in terms of the vierbein  $e_\mu^A$  as follows;

$$B_{\mu\nu}^{AB}(x) = \frac{1}{2} \epsilon^{ABCD} e_\mu^C e_\nu^D, \quad (2)$$

we have the Einstein gravity in the first order formalism. The equation of motion for the spin connection,  $\nabla_{[\mu}^{AB} e_{\nu]}^B = 0$ , can be solved in terms of the vierbein, giving the usual Einstein gravity. The original ‘‘BF’’ action Eq.(1) is invariant under the local Lorentz group of  $O(4)$ , but it has the additional symmetry;

$$B_{\mu\nu}^{AB} \rightarrow B_{\mu\nu}^{AB} + \nabla_{[\mu}^{AC} \Lambda_{\nu]}^{CB}, \quad (3)$$

which is the non-Abelian version of the Kalb-Ramond symmetry generated by the vector-like parameter  $\Lambda_\nu^{BC}$  [4]. Owing to this additional symmetry the ‘‘BF’’ theory becomes topological one [5]. Under the fixing of Eq.(2), the Kalb-Ramond symmetry is broken and the Einstein gravity comes out.

Therefore, the understanding of the transition from the topological theory to the Einstein gravity cannot be realized without catching the physical

meaning of the constraint in Eq.(2). Being not satisfied with the transition done by hand, we are tempting to persue possible mechanisms of generating the constraint dynamically. This is the motivation and the target of our paper. In this paper we will propose the condensation mechanism of the extended objects (string fields) in order to violate the Kalb-Ramond symmetry and to generate the constraint condition. Another scenario is investigated in a separate paper [6], where the Kalb-Ramond symmetry is broken radiatively to generate the metric.

## 2 Chiral decomposition

From the gauge theoretical viewpoint it is helpful to perform the  $SU(2) \times SU(2)$  decomposition of the  $O(4)$  local Lorentz group. This makes the action Eq.(1) into the following form:

$$S = \int \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \left( B_{\mu\nu}^a(x) R_{\lambda\rho}^a(x) + \bar{B}_{\mu\nu}^a(x) \bar{R}_{\lambda\rho}^a(x) \right). \quad (4)$$

Here the first and the second terms represent, respectively, the self-dual and anti-self-dual components;

$$\begin{aligned} B_{\mu\nu}^a &\equiv B_{\mu\nu}^{a4} + \frac{1}{2} \epsilon^{abc} B_{\mu\nu}^{bc} \equiv \frac{1}{2} \eta_{BC}^a B_{\mu\nu}^{BC}, \\ \bar{B}_{\mu\nu}^a &\equiv -\bar{B}_{\mu\nu}^{a4} + \frac{1}{2} \epsilon^{abc} \bar{B}_{\mu\nu}^{bc} \equiv \frac{1}{2} \bar{\eta}_{BC}^a \bar{B}_{\mu\nu}^{BC}, \\ R_{\mu\nu}^a &= \partial_{[\mu} \omega_{\nu]}^a + \epsilon^{abc} \omega_{\mu}^b \omega_{\nu}^c, \quad \bar{R}_{\mu\nu}^a = \partial_{[\mu} \bar{\omega}_{\nu]}^a + \epsilon^{abc} \bar{\omega}_{\mu}^b \bar{\omega}_{\nu}^c, \end{aligned} \quad (5)$$

with the corresponding  $SU(2) \times SU(2)$  gauge fields

$$\omega_{\mu}^a \equiv \frac{1}{2} \eta_{BC}^a \omega_{\mu}^{BC}, \quad \bar{\omega}_{\mu}^a \equiv \frac{1}{2} \bar{\eta}_{BC}^a \bar{\omega}_{\mu}^{BC}, \quad (6)$$

where  $\eta_{BC}^a$  and  $\bar{\eta}_{BC}^a$  denote the 't Hooft symbols [11] for self-dual and anti-self-dual decompositions, respectively, and  $a, b$  and  $c = 1 \sim 3$ . Corresponding to the decomposition, the fixing in Eq.(2) becomes

$$B_{\mu\nu}^a = \frac{1}{2} \eta_{BC}^a e_{\mu}^B e_{\nu}^C, \quad \bar{B}_{\mu\nu}^a = \frac{1}{2} \bar{\eta}_{BC}^a \bar{e}_{\mu}^B \bar{e}_{\nu}^C, \quad (7)$$

which lead to the following relations;

$$\epsilon^{\mu\nu\lambda\rho} B_{\mu\nu}^a B_{\lambda\rho}^b = 2e(x)\delta^{ab}, \quad (8)$$

$$\epsilon^{\mu\nu\lambda\rho} \bar{B}_{\mu\nu}^a \bar{B}_{\lambda\rho}^b = 2e(x)\delta^{ab}, \quad (9)$$

$$\epsilon^{\mu\nu\lambda\rho} B_{\mu\nu}^a \bar{B}_{\lambda\rho}^b = 0. \quad (10)$$

The relations in Eqs.(8) and (9) give the constraints on the traceless symmetric part of the indices  $(a, b)$ , which is the isospin-2 contribution in the composition of the two isospin-1 components  $a$  and  $b$ , namely

$$\epsilon^{\mu\nu\lambda\rho} \left( B_{\mu\nu}^a B_{\lambda\rho}^b - \frac{1}{3} \delta^{ab} B_{\mu\nu}^c B_{\lambda\rho}^c \right) = 0, \quad (11)$$

$$\epsilon^{\mu\nu\lambda\rho} \left( \bar{B}_{\mu\nu}^a \bar{B}_{\lambda\rho}^b - \frac{1}{3} \delta^{ab} \bar{B}_{\mu\nu}^c \bar{B}_{\lambda\rho}^c \right) = 0. \quad (12)$$

The important point here is that if we impose the three constraints in Eqs.(10), (11) and (12), then we can obtain Eq.(7) as the general solutions, by introducing an arbitrary function  $e_\mu^A(x)$  which will afterwards be identified with the vierbein [2]. If we restrict ourselves only to the self-dual component of the anti-symmetric tensor fields  $B_{\mu\nu}^{AB}$  by setting  $\bar{B}_{\mu\nu}^a = 0$ , then we have only one constraint to be analyzed, namely Eq.(11). Therefore, If we can find the dynamical origin of this constraint, we will surely be very close to the dynamical generation of the metric or the phase transition from the topological theory to the Einstein gravity. The same consideration can be done for the anti-self dual component also. Introducing a Lagrange multiplier field  $\phi^{ab}(x)$  with traceless isospin indices, we have the following (chiral) action for the self-dual part:

$$S = \int \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \left( B_{\mu\nu}^a(x) R_{\lambda\rho}^a(x) + \underbrace{\phi^{ab}(x) B_{\mu\nu}^a(x) B_{\lambda\rho}^b(x)}_{\text{constraint term}} \right). \quad (13)$$

Now it is clearly recognized that without the constraint term, the action Eq. (13) is invariant under both (i) the  $SU(2)$  local Lorentz transformation

$$\begin{cases} \omega_\mu^a & \rightarrow \omega_\mu^a + \epsilon^{abc} \omega_\mu^b \Lambda^c - \partial_\mu \Lambda^a \\ B_{\mu\nu}^a & \rightarrow B_{\mu\nu}^a + \epsilon^{abc} B_{\mu\nu}^b \Lambda^c \end{cases} \quad (14)$$

and (ii) the Kalb-Ramond symmetry

$$\begin{cases} \omega_\mu^a & \rightarrow \\ B_{\mu\nu}^a & \rightarrow B_{\mu\nu}^a + \nabla_\mu^{ab} \Lambda_\nu^b - \nabla_\nu^{ab} \Lambda_\mu^b \end{cases} \quad (15)$$

where  $\nabla_\mu^{ab}$  is the covariant derivative of the  $SU(2)$  gauge symmetry,  $\nabla_\mu^{ab} \Lambda_\nu^b = \partial_\mu \Lambda_\nu^a + \epsilon^{abc} \omega_\mu^b \Lambda_\nu^c$ .

A very naive estimation of the degrees of freedom of  $\omega_\mu^a$  and  $B_{\mu\nu}^a$  in this theory is 30, whereas the number of the independent gauge transformations is 15 each of which kills two degrees of freedom, so that the theory without the constraint (“BF theory”) has no local degrees of freedom, or is the topological theory. The rigorous proof on this point can be found in Ref.[5]. The Kalb-Ramond symmetry is, however, broken for the action called “2-form gravity” in Eq.(13), where the constraint Eq.(11) is added to the original “BF” action.

### 3 String theory without the metric (Abelian case)

We may remember that the Kalb-Ramond symmetry was originally introduced as a gauge symmetry of the string theory [4], where the anti-symmetric tensor field  $B_{\mu\nu}^a(x)$  plays the role of the *gauge field* for the string field  $\Psi[C]$  which is a functional defined on a closed curve  $C$ . In the Abelian case the Kalb-Ramond transformation is given by

$$\begin{cases} B_{\mu\nu} & \rightarrow B_{\mu\nu} + \partial_\mu \Lambda_\nu - \partial_\nu \Lambda_\mu \\ \Psi[C] & \rightarrow \exp(i \oint_C dx^\mu \Lambda_\mu(x)) \Psi[C], \end{cases} \quad (16)$$

where  $\Lambda_\mu(x)$  is the parameter of the transformation. If we define  $\delta C^{\mu\nu}(x)$  to be the rectangular deformation of a curve  $C$  at  $x$  in the  $\mu\nu$  direction, the derivative of the string field can be defined as

$$\frac{\delta}{\delta C^{\mu\nu}(x)} \equiv \lim_{\text{area of } \delta C^{\mu\nu} \rightarrow 0} \left( \frac{\Psi[C + \delta C^{\mu\nu}(x)] - \Psi[C]}{\text{area of } \delta C^{\mu\nu}} \right) \quad (17)$$

The covariant derivative is given by

$$\frac{D}{DC^{\mu\nu}(x)} \equiv \frac{\delta}{\delta C^{\mu\nu}(x)} - i B_{\mu\nu}(x), \quad (18)$$

since it satisfies

$$\begin{aligned} & \left( \frac{\delta}{\delta C^{\mu\nu}(x)} - iB'_{\mu\nu}(x) \right) \Psi'[C] \\ &= \exp(i \oint_C dx^\mu \Lambda_\mu(x)) \left( \frac{\delta}{\delta C^{\mu\nu}(x)} - iB_{\mu\nu}(x) \right) \Psi[C] . \end{aligned} \quad (19)$$

Now we have an action invariant under the Kalb-Ramond transformation;

$$\begin{aligned} S &= \int d^4x \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} B_{\mu\nu}(x) R_{\lambda\rho}(x) \\ &+ \sum_C \sum_{x \in C} \epsilon^{\mu\nu\lambda\rho} \left[ \left( \frac{\delta}{\delta C^{\mu\nu}(x)} - iB_{\mu\nu}(x) \right) \Psi[C] \right]^\dagger \left[ \left( \frac{\delta}{\delta C^{\lambda\rho}(x)} - iB_{\lambda\rho}(x) \right) \Psi[C] \right] \\ &+ \sum_C \mathcal{V}[\Psi[C]^\dagger \Psi[C]] \end{aligned} \quad (20)$$

It is the characteristic point that this action is defined using the  $\epsilon$  symbol without using the metric. Therefore the theory may be a topological one, where the configuration of the shape of strings may be analyzed even in this theory with the  $\epsilon$  symbol, but the vibration modes of the strings are difficult to be measured without having the length or the energy scale. On the other hand, the string theory defined using the metric  $g^{\mu[\lambda}g^{\nu\rho]}$  in place of the  $\epsilon^{\mu\nu\lambda\rho}$  was studied more than a decade ago as a field theory of ordinary vibrating strings [8][9].

## 4 Condensation of the strings and generation of the metric (Abelian case)

Now, we will consider the condensation of the strings in the string theory given above. Let us first introduce  $\phi(x)$  as

$$\frac{1}{2} \phi(x) \equiv \sum_{C(\ni x)} \Psi[C]^\dagger \Psi[C]. \quad (21)$$

Then we have the following term in the action,

$$S_C = \int d^4x \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \phi(x) B_{\mu\nu}(x) B_{\lambda\rho}(x), \quad (22)$$

which can be seen as the Abelian version of the constraint term in the 2-form gravity, Eq.(13). Therefore, if the vacuum expectation value  $\langle\phi(x)\rangle$  of the field  $\phi(x)$  is vanishing at high energies, then we have no constraint on the anti-symmetric tensor field, that is, we have the topological “BF” theory, whereas if it gives a large vacuum expectation value, then we have the following constraint:

$$\frac{1}{2}\epsilon^{\mu\nu\lambda\rho}B_{\mu\nu}B_{\lambda\rho}=0. \quad (23)$$

The constraint can be viewed as the Abelian version of the constraint in Eq.(11) which is necessary to generate the concept of the metric and which we are going to obtain dynamically. The mechanism is a kind of the Meissner effect, where the vacuum expectation value of the Higgs field prohibits the penetration of the magnetic field;

$$g^{\mu\nu}A_{\mu}A_{\nu}=0. \quad (24)$$

The magnetic vortex condensation in the dual transformed Higgs model induced the following term in the dually transformed action [9],

$$-\frac{1}{4}(1+\phi)B^{\mu\nu}B_{\mu\nu}, \quad (25)$$

which is just the term in Eq.(22), if the  $g^{\mu[\lambda}g^{\nu\rho]}$  is used in the place of  $\epsilon^{\mu\nu\lambda\rho}$ . (Originally the notation  $W_{\mu\nu}$  was used for the present  $B_{\mu\nu}$  [4][9].) The generated term due to the vortex condensation was seen in the original Higgs model as

$$-\frac{1}{4}(1+\phi)^{-1}F^{\mu\nu}F_{\mu\nu}, \quad (26)$$

which gives the anti-Meissner effect or the confinement when the condensation of the string field occurs having the large value of the  $\phi$ , where the  $F_{\mu\nu}$  is the field strength of the electromagnetic field.

## 5 String theory without the metric (non-Abelian case)

In order to understand the gravity, we should generalize the above discussion starting from Eq.(20) to the non-Abelian case. When we write down the non-Abelian version of the transformation in Eq.(16), we should be careful about

the line integral of the transformation parameter  $\Lambda_\mu^a(x)$ , since it has different transformation properties at different positions. In order to have the definite transformation property under the local  $SU(2)$ , we should make all the points on the curve  $C$  be connected to a special point  $x_0$  by the Wilson operator and the different transformation properties at different points be unified to a single one at  $x_0$ . Then, we have the following non-Abelian version of the Kalb-Ramond transformation,

$$\Psi^i[C; x_0] \rightarrow U[C; x_0]_j^i \Psi^j[C; x_0] \quad (27)$$

with

$$U[C; x_0]_j^i \equiv \left[ \exp \left\{ i \oint_C dx^\mu \Lambda_\mu^a(x) W[x \leftarrow x_0]_b^a T_R^b \right\} \right]_j^i \quad (28)$$

where  $\Psi[C; x_0]$  behaves as a local field at  $x_0$  under the local  $SU(2)$  transformation in the representation  $\{R\}$  and the Wilson operator is defined using the adjoint representation  $\{A\}$  ;

$$W_P[x \leftarrow x_0]_b^a = \left[ P \exp i \int_{P(x \leftarrow x_0)} dx^\mu (\Lambda_\mu^c T_A^c) \right]_b^a, \quad (29)$$

Here the special point  $x_0$  and the path  $P(x \leftarrow x_0)$  are chosen to be along the curve  $C$ . (It is also possible to take  $x_0$  outside the curve  $C$  and the path  $P$  not along the curve  $C$ , but the above choice is the simpler one.) Corresponding to the non-Abelian Kalb-Ramond transformation for the “matter string fields” in Eq.(27), we can find the following transformation-property for the “gauge string fields”  $B_{\mu\nu}[C; x, x_0]$  in the matrix notation, by generalizing the non-Abelian Kalb-Ramond transformation for  $B_{\mu\nu}(x)$  in Eq.(15), that is,

$$B_{\mu\nu}[C; x, x_0] \rightarrow U[C; x_0] B_{\mu\nu}[C; x, x_0] U[C; x_0]^{-1} - \frac{\delta U[C; x_0]}{\delta C^{\mu\nu}} U[C; x_0]^{-1}. \quad (30)$$

Both equations (27) and (30) gurantees the existence of the covariant derivative

$$\frac{D}{DC^{\mu\nu}} \equiv \frac{\delta}{\delta C^{\mu\nu}} + B_{\mu\nu}[C; x, x_0] \quad (31)$$

satisfying

$$\frac{D}{DC^{\mu\nu}} \Psi[C; x_0] \rightarrow U[C; x_0] \frac{D}{DC^{\mu\nu}} \Psi[C; x_0]. \quad (32)$$



The transformation of the Kalb-Ramond string field  $B_{\mu\nu}[C; x, x_0]$  in Eq.(30) can be written more explicitly in terms of the components  $B_{\mu\nu}^a[C; x, x_0]$ ,

$$\begin{aligned} B_{\mu\nu}^a[C; x, x_0] \rightarrow & B_{\mu\nu}^a[C; x, x_0] - \epsilon^{abc} \oint_C dy^\nu \Lambda_\nu^{b'}(y) W_C[y \leftarrow x_0]_{b'b} B_{\mu\nu}^c[C; x, x_0] \\ & - i \left\{ (\nabla_\mu \Lambda_\nu(x) - \nabla_\nu \Lambda_\mu(x))^{a'} W_C[x \leftarrow x_0]_{a'}^a \right. \\ & \left. + \int_{y \geq x} dy^\nu \Lambda_\nu^b(y) W_C[y \leftarrow x]_{bb'} (iT_A^c)^{b'}_{a'} R_{\mu\nu}^c(x) W_C[x \leftarrow x_0]^{a'a} \right\}. \end{aligned} \quad (33)$$

This transformation property is rather different from the original non-Abelian version of the Kalb-Ramond transformation in Eq.(15), but if the closed curve  $C$  becomes very small, the transformation approaches to

$$B_{\mu\nu}^a[C; x, x_0] \rightarrow B_{\mu\nu}^a[C; x, x_0] - i(\nabla_\mu \Lambda_\nu - \nabla_\nu \Lambda_\mu)^a, \quad (34)$$

so that in the small  $C$  limit we have recover the original Kalb-Ramond field;

$$B_{\mu\nu}^a(x) = \lim_{C \rightarrow \text{small}} (-i) B_{\mu\nu}^a[C; x, x_0]. \quad (35)$$

Now, we can write down the invariant action under the non-Abelian version of the Kalb-Ramond transformations obtained above in Eqs.(27), (28), (30) and (33):

$$\begin{aligned} S = & \int d^4x \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} B_{\mu\nu}^a(x) R_{\lambda\rho}^a(x) \\ & + \sum_C \sum_{x_0 \in C} \sum_{x \in C} \epsilon^{\mu\nu\lambda\rho} \left[ \left( \frac{\delta}{\delta C^{\mu\nu}(x)} + T^a B_{\mu\nu}^a[C; x, x_0] \right) \Psi[C; x_0] \right]^\dagger \\ & \quad \times \left[ \left( \frac{\delta}{\delta C^{\lambda\rho}(x)} + T^a B_{\lambda\rho}^a[C; x, x_0] \right) \Psi[C; x_0] \right] \\ & + \sum_C \sum_{x_0} \mathcal{V}[\Psi[C; x_0]^\dagger \Psi[C; x_0]]. \end{aligned} \quad (36)$$

## 6 Condensation of the strings and generation of the metric (non-Abelian case)

Having the non-Abelian action given above, if we will define  $\phi^{ab}[C; x_0]$  by

$$\frac{1}{2} \phi^{ab}[C; x_0] \equiv \Psi[C; x_0]^\dagger T_R^a T_R^b \Psi[C; x_0], \quad (37)$$

then we have the following contribution in the action, namely

$$S'_C = \sum_C \sum_{x_0(\in C)} \sum_{x(\in C)} \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \phi^{ab}[C; x_0] B_{\mu\nu}^a[C; x, x_0] B_{\lambda\rho}^b[C; x, x_0]. \quad (38)$$

Now we will give the same discussion as in the Abelian case on the condensation of the string field: If the vacuum expectation value  $\langle \phi^{ab}[C; x_0] \rangle$  of the isospin-2 part (traceless part with respect to  $(a, b)$ ) of the  $\phi^{ab}[C; x_0]$  is vanishing even in the small  $C$  limit, then we have no constraint given in Eq.(11), so that we are still in the topological phase. If the small  $C$  limit of the isospin 2 part of the condensation  $\langle \phi^{ab}[C; x_0] \rangle$  gives, fortunately, large non-vanishing values, then we obtain the required constraint in Eq.(11). Therefore the condensation of the string field, the “matter” for the “Kalb-Ramond gauge field”  $B_{\mu\nu}^a(x)$ , may trigger the phase transition from the topological “BF” theory to the 2-form gravity which is the chiral part of the Einstein gravity and in which the concept of the metric is recovered.

## 7 Discussion with the quark confinement and the polymer physics

We have as yet not enough ability to clarify the dynamics for the above mentioned condensation of the string fields to occur. It is, however, the mechanism itself is very similar to the proof of the quark confinement as was discussed above in the Abelian case [9]. Both mechanisms of the generation of the metric and of the quark confinement are formally interchanged, according to the replacement of the roles of the epsilon symbol  $\epsilon^{\mu\nu\lambda\rho}$  and the product of the metric,  $g^{\mu[\lambda} g^{\nu\rho]}$ . But, the condensation of the string field may commonly underlie the both phenomena. The Coleman-Weinberg mechanism performed before [9] in the study of the quark confinement due to the vortex condensation might be helpful. The importance of the radiative corrections on our problem can also be found in Ref.[6] and Ref.[7]. In the polymer physics (a kind of string theory with some interactions), a similar condensation mechanism of the spin 2 field exists [10]. If we define  $\mathbf{u}$  as a unit vector representing the direction of the monomer, then the expectation value  $\langle u^a u^b \rangle$  ( $a = 1 \sim 3$ ) is non-vanishing when the stress, given by the stress

tensor  $\sigma^{ab}$ , is applied from outside to the polymer solution: We have

$$\begin{aligned}
Q^{ab} &\equiv \langle (u^a u^b - \frac{1}{3} \delta^{ab}) \rangle \\
&\propto (\sigma^{ab} - \frac{1}{3} \delta^{ab} \sigma_c^c) \\
&\propto (\varepsilon^{ab} - \frac{1}{3} \delta^{ab} \varepsilon_c^c),
\end{aligned} \tag{39}$$

where  $Q^{ab}$  is called “directional order parameter” and  $\varepsilon^{ab}$  is the electric permeability of the polymer solution. The last equation gives the optoelasticity, which can be understood as *the generation of the metric or of the distortion of the space* causing the double refraction, or the gravitational lensing effect in our terminology. The dynamical origin of such an interesting phenomena is *the entropy effects* (the effect of summing up all the possible shapes of polymers) as well as *the interactions between the monomers* such as the nematic interaction of the liquid crystals. Therefore, in order to understand the gravity, especially the generation of the metric, we should pursue the condensation mechanism of the extended objects (the string fields) based on the entropy effects as well as the interactions between the portions of the strings (the monomers), following the polymer dynamics.

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